DIFFERENT MODES OF HEATING WHEN HIGH-POWER RADIATION FLUXES INTERACT WITH A MATERIAL

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The gas-dynamic and thermal processes which occur when a high-power flux of laser radiation interacts with a material are investigated. Fluxes for which the sublimation energy can be neglected compared with the thermal and kinetic energy of the vapors formed are considered. The electron thermal conductivity is considered as well as the hydrodynamic dispersion. The properties of different modes of propagation of temperature waves in a moving medium are studied. The case of an infinitely large absorption coefficient is given particular attention.

1. The possibility of attaining temperatures of the order of 10^8°K by concentrating a high-power flux of laser radiation on a small mass of material has been discussed in [1-3]. The overall energy in the pulse in this case is comparatively small. It has been pointed out [1] that the increase in temperature is limited by the electron thermal conductivity.

Analysis of the solution of the gas-dynamic equations taking into account heat transfer by radiation and thermal conductivity shows that two qualitatively different modes of propagation of heat in a moving medium exist, namely, the so-called TW-I (a temperature wave of the first kind) and TW-II (a temperature wave of the second kind).

The existence of two types of temperature waves was first pointed out in [4]. In [5], which is devoted to a self-similar solution of the one-dimensional plane problem of the motion of a piston in an ideal heat-conducting gas, there is a detailed study of the properties of TW-I and TW-II. The different modes of heating are also considered in [6-9].

2. We will construct a number of plane one-dimensional self-similar problems of radiation gas-dynamics taking into account nonlinear thermal conductivity, by analyzing which we will obtain the qualitative characteristic of the two types of temperature waves. Despite the relative narrowness of the class of appropriate self-similar solutions, the main properties of the TW-I and TW-II modes which emerge from the analysis are also characteristic of the general case, when the conditions for self-similarity are not satisfied.

We will assume that the thermal conductivity and the absorption factor are power functions of the temperature and density:

$$\varkappa = \varkappa_0 T^a \rho^b, \qquad K = K_0 T^{a_1} \rho^{b_1} \tag{2.1}$$

In particular, for a completely ionized plasma (see [10]) the dimensionless constants in Eq. (2.1) are

$$a = {}^5/_2, \;\; b = 0, \;\; a_1 = -{}^3/_2, \;\; b_1 = 2 \tag{2.2}$$

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The gas is assumed to be ideal with the equation of state

$$P = R\rho T, \quad \varepsilon = RT / (\gamma - 1) \tag{2.3}$$

where γ is the ratio of the specific heats, and R is the universal gas constant.

The set of equations of gas dynamics in the one-dimensional plane approximation, taking the laser radiation and the thermal conductivity into account, has the form

$$\begin{gathered} \frac{\partial v}{\partial t} = -R\frac{\partial}{\partial m}(\rho T), \quad \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \frac{\partial v}{\partial m} \\ \frac{R}{\gamma - 1}\frac{\partial T}{\partial t} + R\rho T\frac{\partial v}{\partial m} = -\frac{\partial W}{\partial m}, \quad W = q + W_e \\ W_e = -\varkappa_0 T^a \rho^{b+1}\frac{\partial T}{\partial m} \end{gathered} \tag{2.4}$$

where m is the Lagrange mass variable, t is the time, v is the velocity, ρ is the density, P is the pressure, T is the temperature, q is the flux density of the radiation, and W_e is the heat flux due to the electron thermal conductivity.

We will consider two problems.

Problem A. The radiation flux is completely absorbed in the region of the boundary of the gas with the vacuum or with the piston (m = 0). In the region m > 0 the heat transfer is purely by thermal conduction:

$$q = \begin{cases} q_0 t^g & \text{for} \quad m = 0 \\ 0 & \text{for} \quad m > 0 \end{cases} \tag{2.5}$$

where q_0 and q are constants.

Problem B. At the boundary m = 0 we are given the radiation flux

$$q(0, t) = q_0 t^g \tag{2.6}$$

In the region m > 0 the following transfer equation holds:

$$\partial q / \partial m = -K_0 T^{a_1} \rho^{b_1 - 1} q \tag{2.7}$$

For both problems we will assume that at the point m = 0 the following conditions are also satisfied:

for the vacuum

$$P(0, t) = 0, \quad W_e(0, t) = 0 \tag{2.8}$$

for the piston

$$v(0, t) = v_0 t^{n_0}, \quad W_e(0, t) = 0 \tag{2.9}$$

The initial conditions for t = 0 for all m > 0 have the form

$$v(m, 0) = 0, \quad T(m, 0) = 0, \quad \rho(m, 0) = \rho_0 \tag{2.10}$$

Analysis shows that the solution of problem A is self-similar if the following conditions are satisfied:

$$g = {}^3/_2 (a - 1), \quad n_0 = {}^1/_3 g \tag{2.11}$$

The solution of problem B is self-similar if conditions (2.11) are satisfied and there is also the following additional relation between the parameters a and a_1 (see [11]):

$$a_1 = {}^1/_2 - a \tag{2.12}$$

If the self-similarity conditions are satisfied [Eq. (2.11) or Eqs. (2.11) and (2.12)], the independent variables m and t and the required functions can be represented in the following form:

$$
\begin{gathered}
m = s q_0^{1/3} \rho_0^{2/3} t^{1+g/3}, \qquad v(m, t) = \alpha(s)\, q_0^{1/3} \rho_0^{-1/3} t^{g/3} \\
\rho(m, t) = \delta(s) \rho_0, \qquad T(m, t) = f(s)\, q_0^{2/3} \rho_0^{-2/3} R^{-1} t^{2g/3} \\
W_e(m, t) = \omega(s) q_0 t^g, \qquad q(m, t) = \varphi(s) q_0 t^g
\end{gathered}
\tag{2.13}
$$

Using Eqs. (2.13), we obtain from Eqs. (2.4) a system of ordinary differential equations of the form

$$
\begin{gathered}
n_0 \alpha - n s \alpha' + (\delta f)' = 0, \quad n s \delta^{-1\prime} + \alpha' = 0 \\
(\gamma - 1)^{-1} (2 n_0 f - n s f') + \delta f \alpha' + (\omega + \varphi)' = 0 \\
\omega = -\lambda f^a \delta^{b+1} f'
\end{gathered}
\tag{2.14}
$$

where

$$
n_0 = g/3, \quad n = 1 + g/3
$$

In addition, for problem A we have

$$
\varphi(0) = 1, \quad \varphi(s) = 0 \quad \text{for} \quad s > 0 \tag{2.15}
$$

and for problem B we have

$$
\varphi(0) = 1, \quad \varphi' = -\sigma f^{a_1} \delta^{b_1 - 1} \varphi \quad \text{for} \quad s > 0 \tag{2.16}
$$

The dimensionless quantities λ and σ have the form

$$
\lambda = \varkappa_0 q^{2(a-1)/3} \rho_0^{b-(2a+1)/3} R^{-(a+1)}, \qquad \sigma = K_0 q_0^{(2a_1+1)/3} \rho_0^{b_1 - 1/3 - 2a_1/3} R^{-a_1} \tag{2.17}
$$

In Eq. (2.14) we have denoted the derivatives with respect to the dimensionless variables by f', α', etc.

For both problems the boundary conditions (2.8)-(2-10) in the dimensionless variables (2.13) have the form:

in the case of a vacuum

$$
\delta(0) = 0 \tag{2.18}
$$

in the case of a piston

$$
\begin{gathered}
\alpha(0) = \alpha_0 = v_0 \rho_0^{1/3} / q_0^{1/3} \\
\omega(0) = 0, \quad \alpha(\infty) = 0, \quad f(\infty) = 0, \quad \omega(\infty) = 0, \quad \delta(\infty) = 1
\end{gathered}
\tag{2.19}
$$

3. We will now consider problem A in more detail. The analysis and construction of some examples of self-similar solutions of problem B are considered in [11]. We note that the case when the medium is assumed to be optically thick and the radiation is described within the framework of radiant thermal conduction also reduces to problem A. This case has been investigated in detail by solving the problem of a piston with given temperature conditions in [5].

We will formulate the main results of the investigations made previously.

It has been shown that the solution of problem A for $a > 0$ has the form of a temperature wave which propagates with finite velocity. The position of the temperature wave front is represented by a dimensionless coordinate $s = s_1$, which satisfies the conditions

$$
\alpha(s_1) = 0, \quad f(s_1) = 0, \quad \omega(s_1) = 0, \quad \delta(s_1) = 1 \tag{3.1}
$$

The position of the wave front $s = s_1$ for given boundary conditions and dimensionless constant λ is found by solving the system of equations (2.14) numerically.

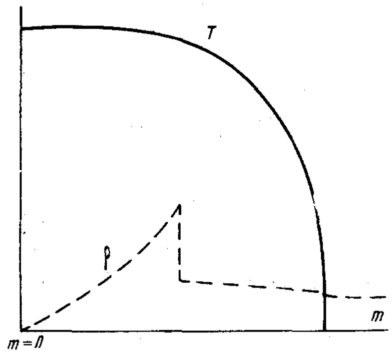

Fig. 1

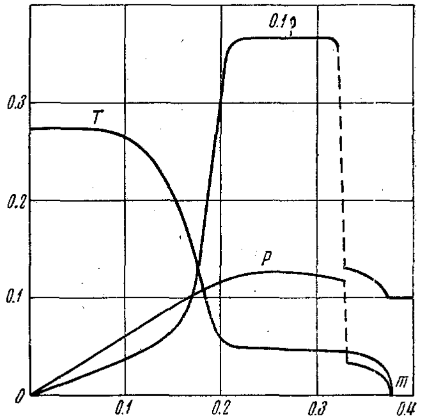

Fig. 2

Analysis of the self-similar solution obtained for different values of the parameters λ shows that two different modes of propagation of heat exist:

1. A temperature wave of the first kind (TW-I) —a mode of propagation of heat with supersonic velocity. This mode has the following properties:

a) the temperature wave propagates with finite velocity with respect to the initial boundary with zero temperature;

b) the velocity of the wave front of TW-I is always greater than the local isothermal velocity of sound $c = \sqrt{RT}$ (supersonic heating);

c) the density and other hydrodynamic quantities behind the front of the TW-I increase;

d) between the piston and the wavefront of TW-I there is an isothermal shock wave.

An example of the supersonic propagation of heat is shown in Fig. 1.

2. A temperature wave of the second kind (TW-II), which is a mode of propagation of heat with subsonic velocity. This mode has the following main properties:

a) the wave front of the TW-II has zero heat flux and maximum density at this point;

b) the velocity of the wave front of TW-II is always less than the local isothermal velocity of sound (subsonic heating);

c) the density and velocity behind the wave front fall (see Fig. 2);

d) the region between the wave front of TW-II and the shock wave which moves ahead of it is almost adiabatic (the heat fluxes are small), the front of the shock wave is somewhat blurred by the thermal conductivity and is an isothermal jump.

A change in the mode of propagation of heat is determined by the values of the parameters λ. It has been shown that when

$$\lambda < \lambda_* \tag{3.2}$$

where λ_* is a certain dimensionless constant, the TW-II mode exists.

When

$$\lambda \geqslant \lambda_* \tag{3.3}$$

the TW-I mode exists.

Using Eqs. (2.17) and the expression for q_0 in terms of the total energy E of the interaction of the source of radiation during the time τ:

$$q_0 = (g + 1)\, E \,/\, \tau^{g+1} \tag{3.4}$$

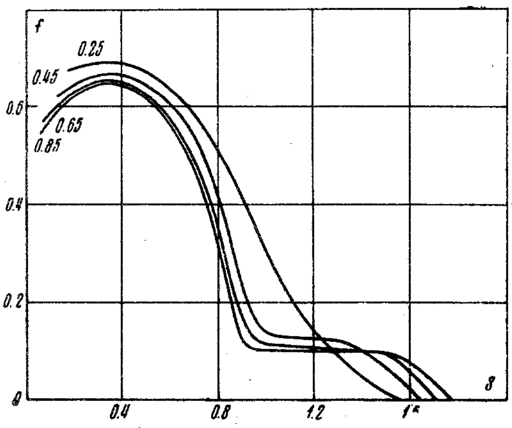

Fig. 3

the left-hand side of the inequalities (3.2) and (3.3) can be written in the form

$$\lambda = \varkappa_0 (g+1)^c E^c \tau^{-c(g+1)} \rho_0^{b-c} R^{-(a+1)} \qquad (3.5)$$

where $c = 2(a-1)/3$.

In particular, it follows from relations (3.2)-(3.4) and Eqs. (2.2) that for the case of a completely ionized plasma (in the case $g=1$) the TW-I mode exists when

$$\frac{E}{\tau^2} > \frac{1}{2}\lambda_* \frac{\rho_0^2 R^{7/2}}{\varkappa_0} \qquad (3.6)$$

and the TW-II mode exists when

$$\frac{E}{\tau^2} \leqslant \frac{1}{2}\lambda_* \frac{\rho_0^2 R^{7/2}}{\varkappa_0} \qquad (3.7)$$

The dimensionless constant λ_* is found by numerical solution of the self-similar problem A.

In this case in relations (3.6) and (3.7) for $\gamma = 5/3$ the value of λ_* is 3.75. However, the gas-dynamic motion becomes of comparatively little importance (the drop in density on the shock wave in the depth of the TW front is not greater than 1.5), and only begins at values of λ greater than 50.

4. We have noted above that problem A is self-similar provided the conditions (2.11) are satisfied. If g is arbitrary (for example, for $q(0, t) = q_0 = \text{const}$), the problem is not self-similar. However, the numerical solution of this problem shows that for the TW-II mode in the heating region, i.e., in the region between the wave front of TW-II and the vapor-vacuum boundary, the solution of problem A in time approaches the self-similar mode, in which the required functions have the form

$$F_i(m, t) = f_i(s) F_{0i} t^{n_i} \qquad (s = m / A_0 t^n)$$

Figure 3 shows a graph of the dimensionless temperature as a function of the self-similar variable s, which illustrates how the solution asymptotically approaches the self-similar mode in the heating region. (Self-similar problems of this type were first considered by Sakharov, Zel'dovich and their co-workers. A similar problem, ignoring the thermal conductivity, was considered in [12, 13]).

In the TW-II mode it can be assumed approximately that the pressure at the front of the temperature wave (P_T) is equal to the pressure at the front of the shock wave (P_V), which moves in front of the TW with respect to the "initial background," with density $\rho(m, 0) = \rho_0$ (see Fig. 2). From the value $P_V = P_T$ is easy to find the mass velocity ($D_V = \sqrt{0.5(\gamma+1)P_T\rho_0}$) and other parameters of the shock wave.

Calculations show that the solution of the problem can be constructed as follows. In the region $0 \leq m \leq m_T$ enveloped by the temperature wave a self-similar solution is constructed from which the pressure on the temperature-wave front $P_T = P(m_T, t)$ and the depth of heating m_T are determined. Using the condition $P_V = P_T$ the self-similar solution is matched to the non-self-similar solution under the existing conditions of the shock wave which moves in front of the temperature wave with respect to the background $\rho = \rho_0$.

The dimensionless variables and the required functions in the heating region can be represented in the following form:

$$\begin{aligned} m &= s q_0^{(2a-b-1)/d} R^{-(2a+1)/d} \varkappa_0^{2/d} t^n \\ v(m,t) &= \alpha(s) q_0^{(1-b)/d} R^{(a+1)/d} \varkappa_0^{-1/d} t^{n_0} \\ T(m,t) &= f(s) q_0^{2(1-b)/d} R^{(1+3b)/d} \varkappa_0^{-2/d} t^{2n_0} \\ \rho(m,t) &= \delta(s) q_0^{2(a-1)/d} R^{-3(a+1)/d} \varkappa_0^{3/d} t^{n_1} \end{aligned} \qquad (4.1)$$

where $d = 2a + 1 - 3b$, and

$$W_l(m, t) = \omega(s) q_0 t^g, \qquad q(m, t) = \varphi(s) q_0 t^g$$

where $\varphi(s) = 1$ for $s = 0$ and $\varphi(s) = 0$ for $s > 0$, and

$$n = 1 + \frac{g}{3} + \frac{2n_1}{3}, \quad n_0 = \frac{g}{3} - \frac{n_1}{3}, \quad n_1 = \frac{2(a-1)g - 3}{2a - 3b + 1}$$

The problem is self-similar for arbitrary values of $g > -1$ (the case when $g \leq 1$ has no physical meaning since in this case the energy of the laser radiation introduced is infinite).

From the self-similar solution one can find numerically the values of the dimensionless coordinate of the wave front of the temperature wave $s = s_1$, where the following conditions are satisfied:

$$f(s_1) = 0, \quad \alpha(s_1) = 0, \quad \omega(s_1) = 0$$

and the dimensionless pressure $\beta(s_1) = \beta_1 > 0$. For example, for the case when $\gamma = 5/3$ when the plasma is completely ionized ($a = 5/2$, $b = 0$) and for a constant boundary flux of radiation ($g = 0$) we have $s_1 = 0.6$ and $\beta_1 = 0.42$.

The depth of heating of the material and the pressure on the heating front are found from the equations

$$m_T = s_1 (\varkappa_0^2 q_0^{2a-b-1} R^{-2(a+1)})^{1/d} t^{g/3+1+2n_1/3} \tag{4.2}$$

$$P_T = \beta_1 (\varkappa_0 q_0^{2(a-b)} R^{-(a+1)})^{1/d} t^{2g/3+n_1/3} \tag{4.3}$$

Assuming further than $p_T = p_V$, we find that the mass velocity and the mass coordinate of the shock wave respectively have the following form:

$$D_v = \sqrt{0.5(\gamma+1)\beta_1}\rho_0^{1/2} (\varkappa_0^{1/2} q_0^{a-b} R^{-(a+1)/2})^{1/d} t^{g/3+n_1/6} \tag{4.4}$$

$$m_v = \frac{\sqrt{0.5(\gamma+1)\beta_1}}{g/3+1+n_1/6} \rho_0^{1/2} (\varkappa_0^{1/2} q_0^{a-b} R^{-(a+1)/2})^{1/d} t^{g/3+1+n_1/6} \tag{4.5}$$

Comparison of the parameters of the shock front and the temperature wave enables us to determine (apart from a dimensionless factor) the critical instant of time at which the self-similar mode is approached. In fact, the difference between the mass coordinates which define the position of the shock front and the temperature wave has the form

$$\Delta m = m_v - m_T = s_1 (\varkappa_0^2 q_0^{2a-b-1} R^{-2(a+1)})^{1/d} t^{g/3+1+n_1/6} (\chi_0 - t^{n_1/2}) \tag{4.6}$$

where

$$\chi_0 = \sqrt{0.5(\gamma+1)\beta_1} (g/3 + 1 + n_1/6)^{-1} s_1^{-1} \rho_0^{1/2} (R^{3(a+1)/2} \varkappa_0^{3/2} q_0^{-(a-1)})^{1/d}$$

Since in the TW-II mode the shock wave moves in front of the temperature-wave front, we must have $\Delta m > 0$.

It follows from Eq. (4.6) that Δm vanishes when $t = 0$ and $t = t_*$, where

$$t_* = (\sqrt{0.5(\gamma+1)\beta_1} (g/3 + n_1/6 + 1)^{-1} s^{-1} \rho_0^{1/2})^{2/n_1} (R^{3(a+1)/2} \varkappa_0^{-3/2} q_0^{1-a})^{2/2a-2g-3)} \tag{4.7}$$

Consequently if $2a + 1 - 3b > 0$, $a > 1$, then when $g < 3(a-1)/2$ the self-similar mode occurs when $t > t_*$, where the "critical" time $t = t_*$ is given by Eq. (4.7).

In particular, the TW-II mode exists at the asymptotic stage of the heating and vaporization process when $g < 0$, i.e., when the flux is very large (infinite) at the initial instant $t = 0$, and then when $t > 0$ decreases sharply following a power law and also when $g = 0$, i.e., in the case when $q(0, t) = q_0 = \text{const}$. At the initial stage of the process the TW-I mode occurs.

When $g > 3(a-1)/2$ the self-similar TW-II mode exists in the initial stage of the process, i.e., when $t < t_*$. In the asymptotic stage the TW-II mode changes into the TW-I mode; i.e., supersonic heating occurs.

In the case when $g=3(a-1)/2$, the self-similar mode occurs both in the heating region and in the shock-wave region. This case has been considered in Sec. 3.

When $\gamma = 5/3$, $g=0$ (constant radiation flux at the boundary), $a = 5/2$, and $b=0$ (completely ionized gas), the temperature T, the depth of heating $m_T = m_1$ and the value $t=t_*$ (the instant when the modes of heat propagation change) have the form

$$T(m, t) = f(s)\, q_0^{1/3} R^{1/6} \varkappa_0^{-2/3} t^{1/3}$$

$$m_1 = 0.6 q_0^{2/3} R^{-7/6} \varkappa_0^{1/3} t^{2/3}$$

$$t_* = \varkappa_0 q_0 \rho_0^{-4} R^{-7/2} (0.4 [0.21 (\gamma + 1)]^{-1/2})^4 \approx {}^1/_{16} \varkappa_0 q_0 \rho_0^{-4} R^{-7/2}$$

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